Welcome to STA 101!

Statistics is a confrontation with uncertainty.

Statistics confronts uncertainty by quantifying it.

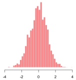
Data analysis

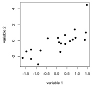
Transforming messy, incomplete, imperfect data into knowledge:

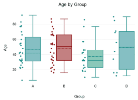
subject	variable_1	variable_2
1	-1.65692830	-2.16524631
2	-0.90396488	-2.97993045
3	1.37141732	0.09720280
4	-0.43176527	0.27970110
5	0.40649190	0.69143221
6	1.47092198	4.47233461
7	-0.78625051	-1.24276055
8	0.64835135	-0.06749005
9	0.06363568	0.33517580

`ggplot` $\Longrightarrow$

`|>` $\Longrightarrow$







	mean	median	std
Variable 1	0.795	-0.292	0.200
Variable 2	0.343	0.616	0.834
Variable 3	1.587	0.508	2.501

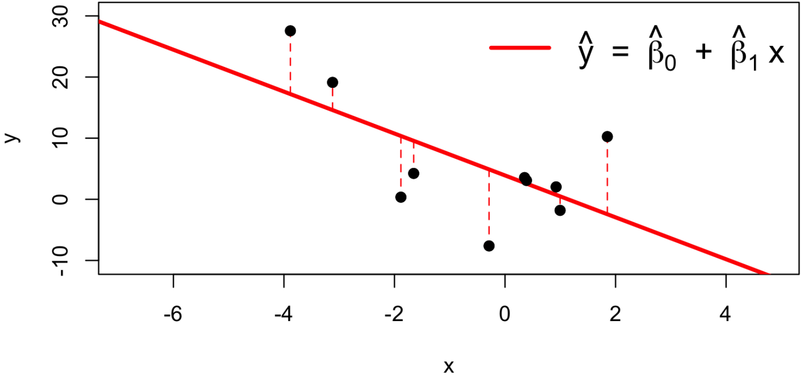
Statistical inference

Quantifying our uncertainty about that knowledge:

- **Question**: What's the number?
- **Answer**: best-guess $\pm$ margin-of-error

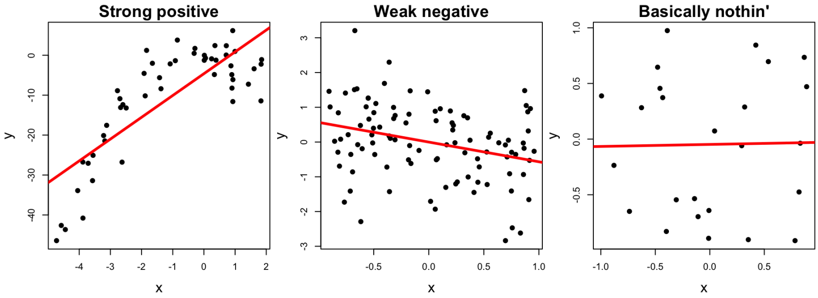
Recap: simple linear regression

Concisely summarize the observed association between two variables using a line of best fit:



The slope and intercept estimates $\hat{\beta}_0$, $\hat{\beta}_1$ are chosen to minimize the sum of squared deviations from the line (residuals).

Interpreting the coefficient estimates in simple regression

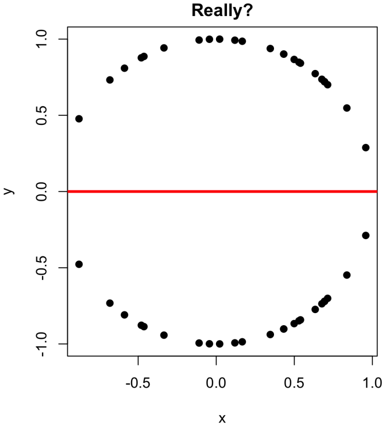


- The sign of $\hat{\beta}_1$ tells you if the variables are positively or negatively correlated (or not at all, if slope $= 0$);

- The magnitude of $\hat{\beta}_1$ tells you something about the strength of the general association.

  Note: The magnitude *does not* tell you about the strength of the correlation per se. I misspoke in this class.

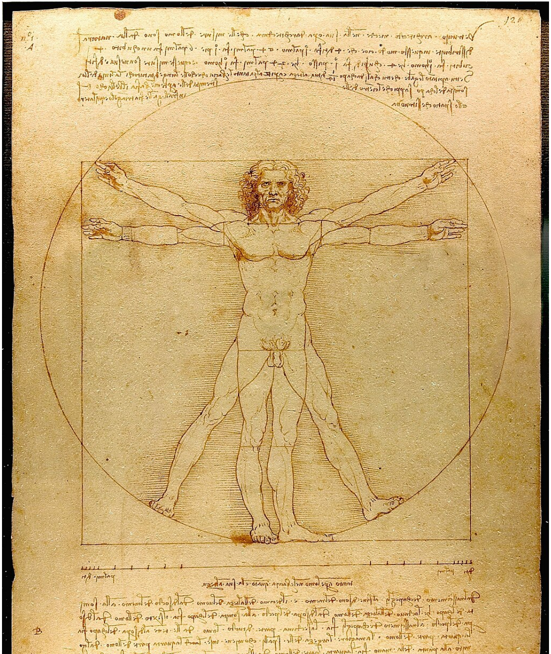
Beware: correlation is not the only kind of association...



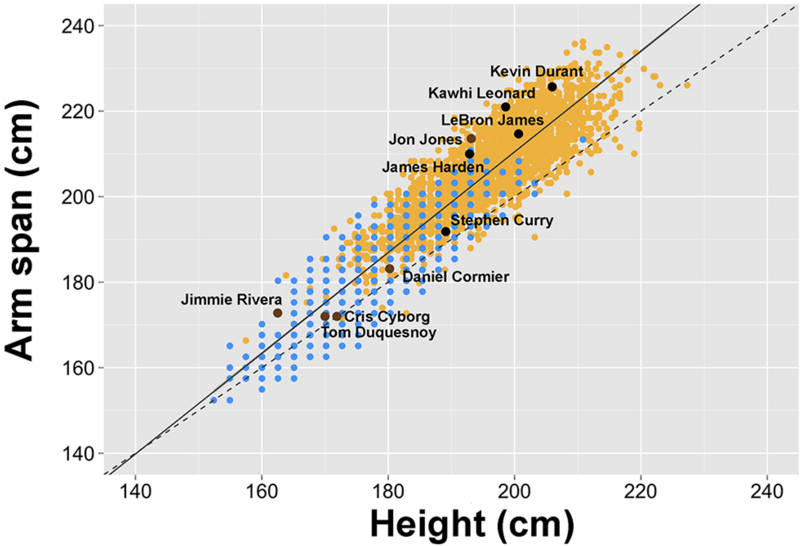
The best fit line says x and y are not correlated. Maybe so, but there is clearly *some* association.

ABV: **A**lways **B**e **V**isualizing

Example: height vs. wingspan



Example: height vs. wingspan



Source: `http://www.jaspe.ac.me/clanci/JASPE_July_2018_Monson_3-8.pdf`

Clarification: notation

Population: The "idealized" linear regression model:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i.$$

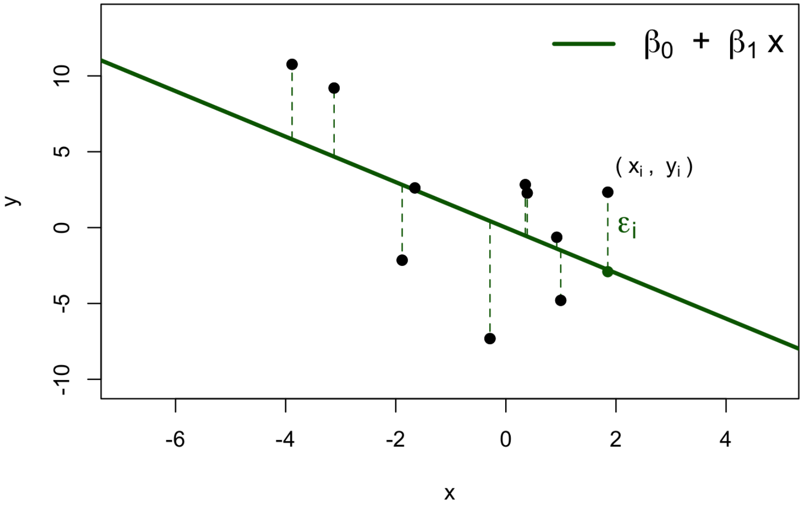
β_0, β_1, and ε_i are unknown. Revealed only with *infinite* data.

Sample: You collect a *finite* sample (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) and calculate the fitted regression model:

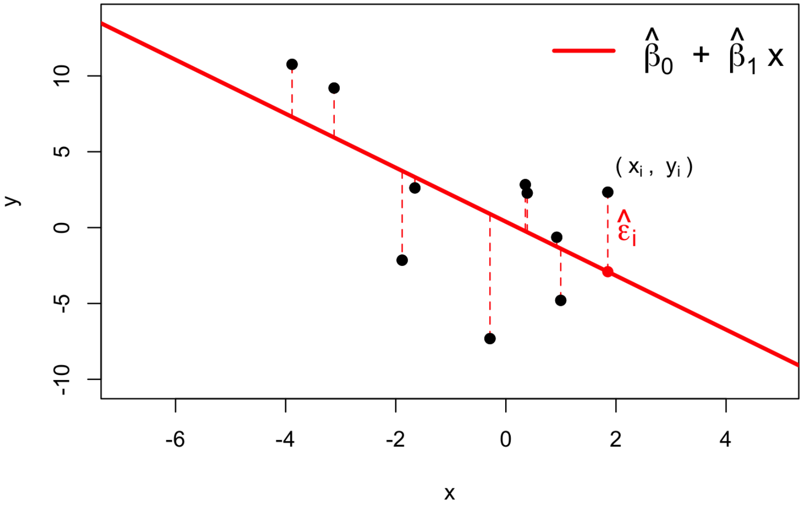
$$\begin{aligned} \hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_i, \\ \hat{\varepsilon}_i &= y_i - \hat{y}_i. \end{aligned}$$

Hopefully: As your sample size n gets bigger, your estimates $\hat{\beta}_0$, $\hat{\beta}_1$ get closer and closer to the ideal, population values β_0, β_1.

The population version



The sample version



Clarification: why squared error?

Why do we minimize

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2,$$

and not

$$\sum_{i=1}^{n}(y_i - \hat{y}_i),$$

or

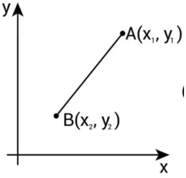
$$\sum_{i=1}^{n}|y_i - \hat{y}_i| \quad ?$$

Why not $\sum_{i=1}^{n}(y_i - \hat{y}_i)$?

Because it's trash.

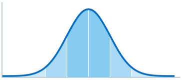
Why choose squared error?

- Squared error is computationally convenient (take my word for it);
- Squared error is intimately related to the *mean*, while absolute error is intimately related to the *median* (take my word for it);
- Squared error plays nice with the geometry of Euclidean space:



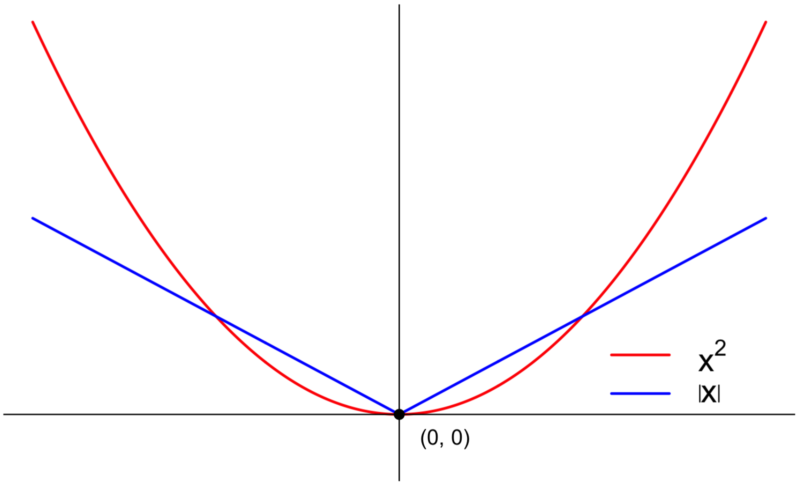
$$d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Squared error plays nice with the *bell curve*:



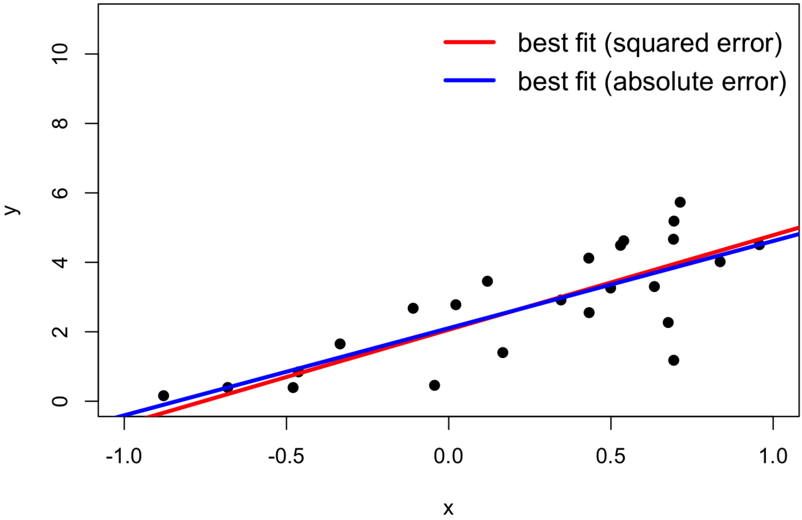
$$p(x) \propto \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

But absolute error is still no joke!

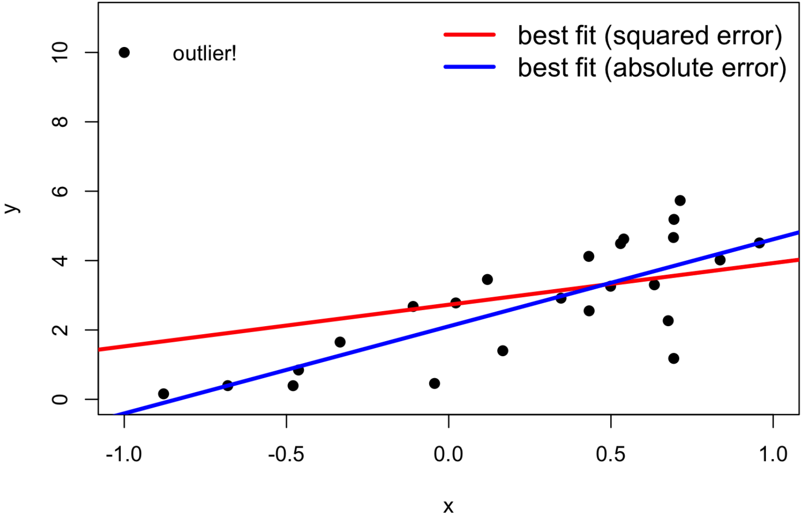


Squared error gives increasingly more weight to data points that are far away from the others (outliers); absolute error is more chill.

"Well-behaved" data: these are basically the same

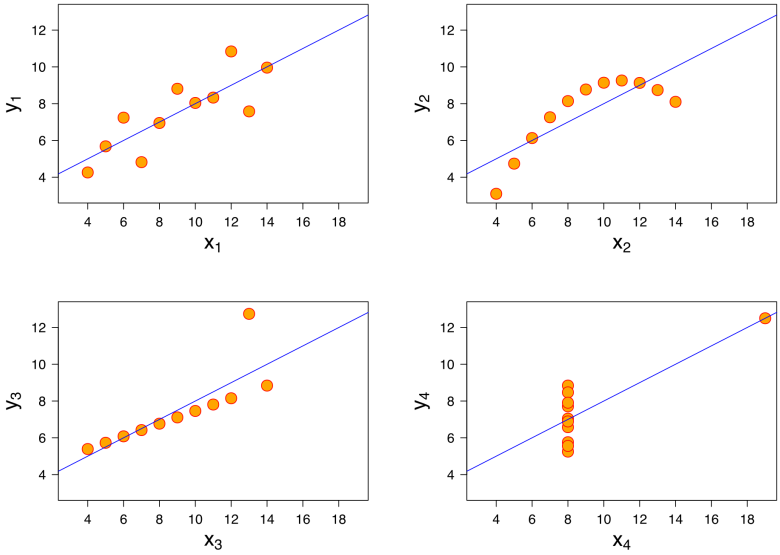


Add a single outlier...



Regression based on absolute error may be “more robust.”

For the upteenth time...



Square vs. absolute error: bottom line

As I said last time, squared error and absolute error are "first class citizens" from a conceptual point of view. They both have pros and cons, and you may prefer one or the other depending on what you are trying to do.

But throughout this course (and most courses you might take), we focus on the squared error version.

Today's topic

Multiple linear regression.

Goal

Study the association between multiple variables (not just two).

Subtext

Assess the causal impact of one variable on another *while accounting for other factors*.

Warning

Association alone does not imply causation.

Linear regression with two predictors

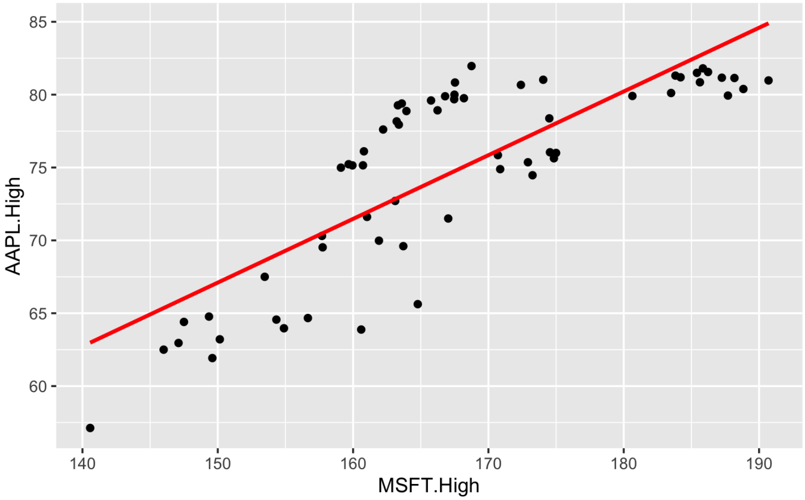
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon.$$

- y: *outcome* or *response* variable;
- x_1, x_2: *predictors*, *covariates*, *regressors*, *features*, ...;
- β_0, β_1, β_2: *coefficients* or *parameters*;
- ε: *error* or *residual*;

This model predicts y given x_1 and x_2.

Recall: stock prices (first quarter of 2020)

$$\widehat{\text{AAPL}} = \underbrace{\hat{\beta}_0}_{1.52} + \underbrace{\hat{\beta}_1}_{0.437} \text{MSFT}$$

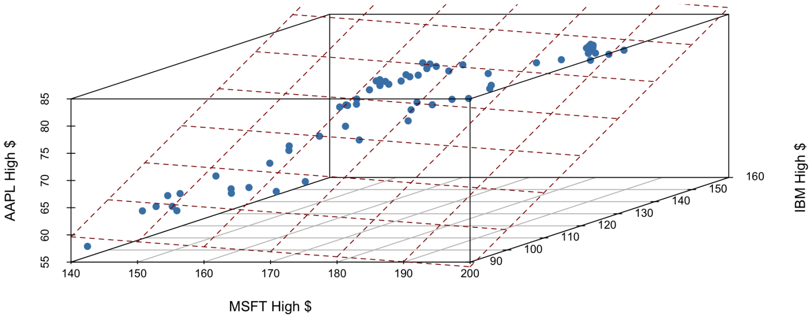


Include a third stock (IBM)

Model:

$$\widehat{\text{AAPL}} = \underbrace{\hat{\beta}_0}_{31.24} + \underbrace{\hat{\beta}_1}_{-0.091} \text{MSFT} + \underbrace{\hat{\beta}_2}_{0.458} \text{IBM}$$

2 predictors + 1 outcome = 3 dimensions:



The line of best fit becomes a **plane** of best fit. Already hard to visualize. Becomes impossible in higher dimensions.

Multiple linear regression

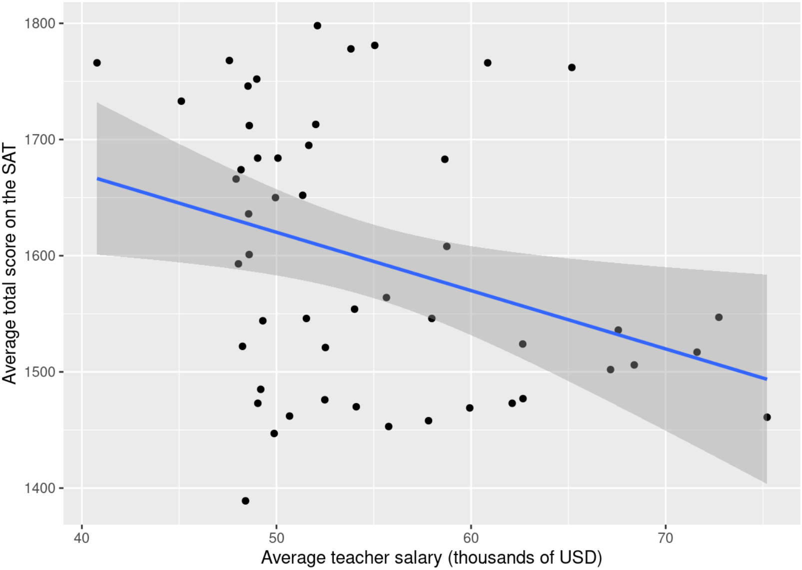
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_p x_p + \varepsilon.$$

- y: *outcome* or *response* variable;
- x_1, x_2, ..., x_p: *predictors*, *covariates*, *regressors*, *features*, ...;
- β_0, β_1, β_2, ..., β_p: *coefficients* or *parameters*;
- ε: *error* or *residual*;

This model predicts y given x_1, x_2, x_3, ..., x_p.

The "concise" numerical summary is a **hyperplane** of best fit, which human beings cannot visualize.

Why do we need more predictors?



Why do we need more predictors?